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*Research Article*

# Effects of Quintessence Dark Energy on the Action Growth and Butterfly Velocity

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In this work we are about to investigate the effects of quintessence dark energy on evolution of the computational complexity relating to the AdS/CFT correspondence. We use "*complexity = action*" conjecture for a charged AdS black hole surrounded by the dark energy at the quintessence regime. Then we try to find some conditions on the quintessence parameters where the Lloyd bound is satisfied in presence of affects of the quintessence dark energy on the complexity growth at the late time approximations. We compare late time approximation of the action growth by perturbed geometry in small limits of shift function. Actually we investigate the evμolution of complexity when thermofield double state on the boundaries is perturbed by local operator corresponding to a shock wave geometry as holographically. Furthermore we seek spread of local shock wave on the black hole horizon in presence of the quintessence dark energy.

## 1. Introduction

In the perspective of gauge/gravity duality evolution of all dynamical fields in the anti-de Sitter (AdS) gravity have dual pictures in the boundary field theories on which the gravity has been removed [1]. Actually this duality acts as a dictionary for all field theory characteristics in the language of black hole physics in AdS spacetime. One of important conjectures in the holographic context is about computational quantum complexity which implies the minimum quantum gates necessary to produce states associated with boundary complexity from the reference state. These conjectures are based on the behavior of a patch created by the light rays emitted from $t_L$ on the left boundary and $t_R$ on the right side of a two-sided eternal black hole, called Wheeler-DeWitt (WDW) patch [2, 3]. The old conjecture states "complexity = volume" (CV), in which the volume of a maximal space-like slice in the black hole interior that connects $t_L$ and $t_R$ supposed to be appropriated with computational complexity in its conformal field theory dual on the boundary [4]. This conjecture is a result of the behavior of the interior volume of black hole which grows linearly with time, so it could be translated with the growth of computational complexity on the boundary with time [5, 6]. However, if the bulk contains a shock wave the interior volume of the black hole shrinks for a finite time interval and shows an opposite behavior. In the new conjecture of "complexity = action" (CA), computational complexity of a holographic state on the boundary pictures is as the on-shell action in WDW patch. This new conjecture has some preferences with respect to the old one and solve some problems which the "CV" conjecture suffers. Lloyd showed [7] the growth rate of quantum complexity has an upper bound which is related to the average energy of the orthogonal quantum states $E$ such that

$$\frac{d(Action)}{dt} \leq 2E. \tag{1}$$

In this new conjecture, authors of the works [2, 3] concluded that the action growth of WDW patch obeys this bound at late time approximation which is provided us to work with orthogonal states. At this approximation we can have a general and universal form of the above bound for a rotating charged black hole as follows [8].

$$\frac{d\mathcal{A}}{dt} \leq \left(M - \Omega J - \mu Q\right)_{+} - \left(M - \Omega J - \mu Q\right)_{-}, \tag{2}$$

where + and − indicate the lowest and highest energy of states. These states for a black hole with multiple horizon happen at the most outer $(r_+)$ and the most inner $(r_-)$ horizons.

In the other side, shock waves near the horizon of an AdS black hole describe chaos in a thermal CFT [9–11] and could have interesting effects on the boundary complexity. From the point of view of holography dictionary the spreading of the shock wave near the horizon has butterfly effect in the boundary field theory. This effect could be seen with a sudden decay after scrambling time $t_* = (\beta/2\pi)\ln S$, in which "$S$" stands for the black hole entropy. It is the necessary time for the black hole as the fastest scramblers to render the density matrix of a small essentially exactly thermal subsystem. The spreading local shock waves in the bulk arise from throwing a few quanta into the horizon which corresponds to perturb thermofield double state $|TFD\rangle$ on the boundaries by local operators. The growth of spreading the shock wave on the boundary is identified with butterfly velocity "$v_B$" could be obtained by solving the equation of motions of perturbed geometry. Complexity growth rate and the effects of butterfly on it, are investigated on various gravity models for the bulk. In refs. [2, 3, 8] the authors investigated action growth for various AdS black holes and tested the Lloyd bound by considering the effects of charge. The growth of holographic complexity is studied in massive gravity in ref. [12], and in a more variety of other works [13–19]. On the other side, some of works have been done about studying the shock wave geometry in different gravity models in the bulk [9–11, 20, 21] and effects of them were investigated on the action growth by obtaining butterfly velocity and comparing them with other simple models [22, 23].

Motivation of studying the effect of dark energy arises from several works in holographic context. For instance Chen et al. found that quintessence dark energy can affect the s-wave and p-wave holographic superconductor [24]. In the other side, Kuang et al. studied the holographic fermionic spectrum dual to AdS black brane in 4 dimensions in the presence of quintessence dark energy and showed that this fermionic system exhibits a nonFermi liquid behavior [25]. So it would be natural to investigate other aspects of holographic effects of this quintessence dark energy such as its impact on complexity growth or its effect on the spread of chaos on boundary. Quintessence dark energy introduced by an equation of state arisen from its energy tensor has a state parameter which is varied like $-1 < \omega < -1/3$. This state parameter which is a factor to explain the accelerating expansion of the universe, could define various regimes and could be fixed by regarding some cosmological observations. Here we like to study effects of this factor on the holographic complexity of a AdS black hole which is perturbed with a shock wave matter field. It can help us to get more profound understanding from the entropy of the black hole in presence of the quintessence dark energy by attention to some related bounds like Lloyd bound. Also it could help us to have more information about quintessence state parameter to get a better understanding of late time acceleration of the universe. Furthermore it could be interesting how the spreading of a chaos could be sensitive for changes of $\omega$. This could give a comprehensive and statistical study for different regimes of the used gravity theory during the evolution of the action. Moreover, from [26] we know free parameter $a$ emerged from quintessence energy tensor should be considered as thermodynamic variable and so its physical insight needs to investigation more. So as next work we are interested to study dual CFT perspective of this variable to approach to goal of this article.

In the present work we consider the effects of quintessence dark energy on the AdS black holes geometry, and therefore we will see how it changes the action growth rate and butterfly velocity in the shock wave geometry. Quintessence dark energy is a canonical scalar field which is one of the successful theories to explain the acceleration phase of the universe [27–29]. In this model which was first introduced by Kiselev [30] an additional energy-momentum tensor of quintessence counterpart must be added to the Einstein equation as $\mathcal{G} = \kappa\left(T_{matter} + T_{\text{quintessence}}\right) = \kappa\mathcal{T}$. The effects of quintessence have been studied in a wide range of works and thermodynamics of the various black holes have been investigated when they are surrounded by the dark energy [30–35]. It would be challenging to see how it affects the holographic characteristics as well. Layout of this work is as follows.

We first study the action growth in the presence of quintessence dark energy in Section 2. Then we discuss about conditions where the Lloyd bound [7] could be hold with new charge associated with this new field. In Section 3 we calculate the butterfly velocity related to the spreading of perturbation and compare the action growth in the presence of a local shock wave geometry in the gravity model under consideration. Section 4 denotes to concluding remarks and outlook of the work.

## 2. The Rate of Action Growth in Presence of the Dark Energy

We consider RN-AdS black hole surrounded by the quintessence dark energy in four dimensional curved space time. It could be described by the following action functional.

$$\mathcal{S} = \mathcal{S}_{bk} + \mathcal{S}_{bd}. \tag{3}$$

where the first part is related with the bulk action contains Einstein-Maxwell Lagrangian density defined in the AdS spacetime as follows.

$$\mathcal{S}_{bk} = \frac{1}{16\pi G}\int_{bulk} d^4x\left(\sqrt{-g}\left[R - 2\Lambda - F^{\mu\nu}F_{\mu\nu}\right] + \mathcal{L}_q\right). \tag{4}$$

In the above action the cosmological constant is related to the AdS space radius $L$ by $\Lambda = -3/L^2$ in four dimension. The second term in the action (4), $\mathcal{L}_q$ implies on the lagrangian of the quintessence dark energy as a barotropic perfect fluid defined by [36]

$$\mathcal{L}_q = -\rho c^2\left(1 + \omega\ln\left(\frac{\rho}{\rho_0}\right)\right), \tag{5}$$

in which $c$ is the light speed and $\rho_0$ is integral constant which is come from singularity cut-off. Also the quintessence dark energy barotropic index satisfies $-1 < \omega < -1/3$. It comes from the quintessence dark energy equation of state as $p = \omega\rho c^2$ in which $\rho$ is energy density and $p$ is corresponding isotropic pressure.

In the other side the Gibbons-Hawking-York (GHY) boundary part of the action term within the WDW patch at the late time approximation is defined by

$$\mathcal{S}_{bd} = \frac{1}{8\pi G}\int_{\text{boundary}} d^3x\,\sqrt{-h}K, \tag{6}$$

where $h$ stands for the determinant of induced metric on the boundary of AdS bulk and $K$ represents the trace of extrinsic curvature. We should note the authors of the work [37] showed that "CA" conjecture suffers from some ambiguities related to the null surface's parametrization and they found joint and boundary terms which are absent in this proposal. However authors of the works [38, 39] proved that the action growth at late time approximation does not need these extra terms. So for the present work, all other boundary terms and joint terms vanish at late time approximation.

In general, a line element for spherically symmetric static geometry in the Schwarzschild coordinates could be given by

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\vec{x}_2^{\,2}, \tag{7}$$

with the following solution in presence of the quintessence dark energy effect [30].

$$f(r) = 1 - \frac{2M}{r} - \frac{\Lambda r^2}{3} + \frac{q_E^2}{r^2} - \frac{a}{r^{3\omega+1}}, \tag{8}$$

where the positive constant "$a$" treats as normalization factor for density of the quintessence dark energy via $\rho = -3a\omega/2r^{3(\omega+1)}$. The electromagnetic tensor field $F_{\mu\nu}$ is defined by 4-vector potential $A_\mu$ as $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$. This is a gauge field and so for a spherically symmetric static metric (7) we can take its form as spherically symmetric time independent function for simplicity reasons as follows.

$$A = A_t dt = -q_E\left(\frac{1}{r} - \frac{1}{r_+}\right)dt. \tag{9}$$

Substituting the metric solution (7) with (8) one can calculate the Ricci scalar as

$$R = 4\Lambda + \frac{3a\omega(3\omega-1)}{r^{3(\omega+1)}}. \tag{10}$$

To study the evolution of WDW action of this model it is useful to depict its Penrose diagram at late time approximation. As the black hole solution (7) has multiple horizons so we must consider its behavior between the lowest and the highest energy which happen at $r_-$ and $r_+$, respectively. In Figure 1, we depicted the evolution of WDW patch for black hole containing the multiple horizons at late time approximation [40]. By increasing the time on the boundary the patch terminates at location of $r = r_{meet}(t_L, t_R)$ for all charged black holes. Time transition says us that the action growth at late time approximation only relates to the dark blue region behind the future horizon in Figure 1. Of course, the tiny part above the meet line is in second order $\delta^2$ which is negligible.

Now by studying time dependence behavior of total action (3) in the quintessence regime of dark energy, we can evaluate holographic complexity growth. By considering the Ricci scalar (10) and the quintessence lagrangian density (5) one can calculate the action growth of the bulk such that

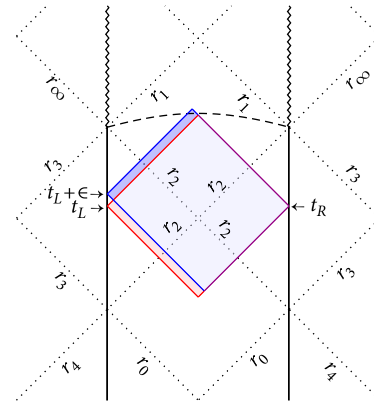


Figure 1: Penrose diagram for a two sided black hole with multiple horizons in which WDW patch evolves at late time approximation and terminates at meet point with $t_L = t_R$ and $t = 0$. $r_0$ is the null spatial infinity and the wavy lines indicate the singularities at $r = 0$, also $r_\infty$ stands for $r = -\infty$.

$$\frac{d\mathcal{S}_{bk}}{dt} = \frac{1}{16\pi G}\int\int_{r_-}^{r_+} r^2\left(2\Lambda + \frac{3a\omega(3\omega-1)}{r^{3(\omega+1)}} + \frac{2q_E^2}{r^4} + \lambda(\omega)\frac{1}{r^{3\omega+1}} + \mu(\omega)\frac{\ln(r)}{r^{3\omega+1}}\right)dr\,d\Omega_2, \tag{11}$$

in which

$$\lambda(\omega) = -\frac{3a\omega}{2}\left(1 + \omega\ln(-\omega)\right),\qquad \mu(\omega) = \frac{9a\omega^2(\omega+1)}{2}, \tag{12}$$

and we put $\rho_0 \equiv 3a/2$. It is simple to show $\lambda(\omega) > 0$ and $\mu(\omega) > 0$ in quintessence regime $-1 < \omega < -1/3$ as follows. By setting $\Omega_2/4\pi G = 1$ (11) reads

$$\begin{aligned}\frac{d\mathcal{S}_{bk}}{dt} = &-\frac{1}{2L^2}\left(r_+^3 - r_-^3\right) - \frac{q_E^2}{2}\left(\frac{1}{r_+} - \frac{1}{r_-}\right)\\ &- \frac{a(3\omega-1)}{4}\left(\frac{1}{r_+^{3\omega}} - \frac{1}{r_-^{3\omega}}\right)\\ &- \frac{1}{12\omega}\left(\lambda + \frac{\mu}{3\omega}\right)\left(\frac{1}{r_+^{3\omega}} - \frac{1}{r_-^{3\omega}}\right)\\ &- \frac{\mu}{12\omega}\left(\frac{\ln(r_+)}{r_+^{3\omega}} - \frac{\ln(r_-)}{r_-^{3\omega}}\right).\end{aligned} \tag{13}$$

It is also easy to see $(\lambda + (\mu/3\omega)) > 0$ for quintessence regime $-1 < \omega < -1/3$.

To obtain the action growth of the boundary part we must evaluate the extrinsic curvature associated to the metric solution (7) for which we have

$$K = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\sqrt{f(r)}\right) = \frac{2}{r}\sqrt{f(r)} + \frac{f'(r)}{2\sqrt{f(r)}}, \tag{14}$$

where the prime $\prime$ notes to derivative with respect to "$r$". By this definition time derivative of the second part of the action (3) leads to the following form.

$$\begin{aligned}\frac{d\mathcal{S}_{bd}}{dt} &= \frac{1}{8\pi G}\int_{\text{boundary}} d\Omega_2\left(\sqrt{-h}K\right)\\ &= \left[rf(r) + \frac{r^2 f'(r)}{4}\right]_{r_-}^{r_+}\\ &= \left(r_+ - r_-\right) + \frac{3}{2L^2}\left(r_+^3 - r_-^3\right) + \frac{q_E^2}{2}\left(\frac{1}{r_+} - \frac{1}{r_-}\right)\\ &\quad + \frac{3a(\omega-1)}{4}\left(\frac{1}{r_+^{3\omega}} - \frac{1}{r_-^{3\omega}}\right).\end{aligned} \tag{15}$$

Adding (13) and (15) we obtain total growth action for quintessence RN-AdS black hole such as follows.

$$\begin{aligned}\frac{d\mathcal{S}}{dt} &= \left(r_+ - r_-\right) + \frac{r_+^3 - r_-^3}{L^2} - \frac{a}{2}\left(\frac{1}{r_+^{3\omega}} - \frac{1}{r_-^{3\omega}}\right)\\ &\quad - \frac{1}{12\omega}\left(\lambda + \frac{\mu}{3\omega}\right)\left(\frac{1}{r_+^{3\omega}} - \frac{1}{r_-^{3\omega}}\right)\\ &\quad - \frac{\mu}{12\omega}\left(\frac{\ln(r_+)}{r_+^{3\omega}} - \frac{\ln(r_-)}{r_-^{3\omega}}\right).\end{aligned} \tag{16}$$

Solving the horizon equation $f(r_\pm) = 0$ one can obtain the following expressions for charge and mass of the RN-AdS black hole.

$$q_E^2 = r_+ r_-\left[1 + \frac{1}{L^2}\frac{r_+^3 - r_-^3}{r_+ - r_-} - a\frac{r_+^{-3\omega} - r_-^{-3\omega}}{r_+ - r_-}\right], \tag{17}$$

$$\begin{aligned}M &= \frac{1}{2}\left[\left(r_+ + r_-\right) + \frac{1}{L^2}\frac{r_+^4 - r_-^4}{r_+ - r_-}\right.\\ &\quad \left. + \frac{a}{2}\frac{r_+}{r_-^{3\omega}\left(r_+ - r_-\right)}\left(\frac{r_-}{r_+} - \left(\frac{r_-}{r_+}\right)^{3\omega}\right)\right].\end{aligned} \tag{18}$$

By attention to these expressions the total growth rate (17) could be rewritten as follows.

$$\begin{aligned}\frac{d\mathcal{S}}{dt} &= -q_E^2\left(\frac{1}{r_+} - \frac{1}{r_-}\right) + \frac{a}{2}\left(\frac{1}{r_+^{3\omega}} - \frac{1}{r_-^{3\omega}}\right)\\ &\quad - \frac{1}{12\omega}\left(\lambda + \frac{\mu}{3\omega}\right)\left(\frac{1}{r_+^{3\omega}} - \frac{1}{r_-^{3\omega}}\right)\\ &\quad - \frac{\mu}{12\omega}\left(\frac{\ln(r_+)}{r_+^{3\omega}} - \frac{\ln(r_-)}{r_-^{3\omega}}\right).\end{aligned} \tag{19}$$

Looking to the works presented by Brown et al. [2, 3], one can infer there are some extra terms due to the presence of quintessence dark energy. If we rewrite this expression with respect to thermodynamic variables we find

$$\begin{aligned}\frac{d\mathcal{S}}{dt} &= \left(M - \mu_+ q_E - \mathcal{A}_+ a\right) - \left(M - \mu_- q_E - \mathcal{A}_- a\right)\\ &\quad - \frac{1}{12\omega}\left(\lambda + \frac{\mu}{3\omega}\right)\left(\frac{1}{r_+^{3\omega}} - \frac{1}{r_-^{3\omega}}\right)\\ &\quad - \frac{\mu}{12\omega}\left(\frac{\ln(r_+)}{r_+^{3\omega}} - \frac{\ln(r_-)}{r_-^{3\omega}}\right),\end{aligned} \tag{20}$$

in which $\mu_\pm = q_E/r_\pm$ stands for chemical potential, $\mathcal{A}_\pm = -1/2r_\pm^{3\omega}$ is conjugated potential for parameter "$a$". As we expect for $a = 0$ the second line in the above result vanishes. If we take "$E$" for the average energy of the quantum states then the rate of quantum complexity satisfies the Lloyd bound [7] as

$$\frac{d\mathcal{C}}{dt} \le \frac{2E}{\pi\hbar}. \tag{21}$$

This satisfaction arises from the conditions $\mu > 0, \lambda > 0$ and $\lambda + \mu/3\omega > 0$ which are mentioned in the above for the quintessence regime $-1 < \omega < -1/3$.

## 3. Butterfly Effect with Shock Wave Geometry

The shock wave geometry happens when our black hole solution is perturbed by a small amount of energy. Study of the shock wave geometry can be done by calculating the butterfly velocity which is the velocity of shock wave near the horizon. To do so we first rewrite the black hole solution (7) in the Kruskal coordinates system such that

$$ds^2 = -2H(u,v)du\,dv + h(u,v)d\overrightarrow{x}_2^{\,2}, \tag{22}$$

where we defined

$$H(u,v) = -\frac{4}{uv}\frac{f(r)}{\left[f'(r_h)\right]^2}, \tag{23}$$

and $h(u,v) = r^2$ where and from now on we mark outer horizon by $r_h$ instead of $r_+$ for simplicity reasons. As we know that there are the following relationship between the null Kruskal coordinates and the spherical coordinates.

$$\begin{aligned}u &= e^{(2\pi/\beta)\left(-t + r_*(r)\right)},\\ v &= -e^{(2\pi/\beta)\left(t + r_*(r)\right)},\end{aligned} \tag{24}$$

with the thermodynamic parameter $\beta = 1/k_B T$ in which $T$ is temperature and $k_B$ is the Boltzmann constant. Also $r_* = \int dr/f(r)$ called as the tortoise spatial radial coordinate. For neighborhood of the exterior horizon $r_h$ the tortoise coordinate is approximated with the following form.

$$r_* \approx \frac{1}{f'(r_h)}\ln\left(\frac{r - r_h}{r_h}\right) + \cdots. \tag{25}$$

Now by rewriting the metric (22) in the new coordinates we can study the effects of disturbance as a shock wave geometry. Actually when a scalar operator $w$ acts on the boundary at $t_w < 0$ this shock wave creates. If $t_w$ be large enough then this shock creates a particle of null matter which travels along $u = 0$ in the bulk. Suppose that the metric has form like (22) for $u < 0$ but it is changed to a perturbed metric in which $v$ is replaced by $v + \alpha\left(x^i\right)$ [8]. $\alpha\left(x^i\right)$ is called the (red) shift function which shows a boundary perturbation in the direction of $x^i$. This shift function creates some similarities for WDW patch with unperturbed geometry at late time approximation which is studied in previous section. In fact when the shift function takes some large values then the light rays of WDW patch run into the past singularity which is similar to early time approximation, since for small shift function similar to the late time approximation, these light rays meet each other behind the past horizon [3]. Applying some new transformations as

$$\begin{aligned} U &= u, \\ V &= v + \theta(u)\alpha\left(x^i\right), \\ X^i &= x^i, \end{aligned} \tag{26}$$

in which $\theta(u)$ represents the Heaviside step function, the metric line element (22) then takes the new form as follows.

$$\begin{aligned} ds^2 &= -2H(U,V)dUdV + h(U,V)d\vec{x}_2^{\,2} \\ &\quad + 2H(U,V)\alpha\left(X^i\right)\delta(U)dU^2, \end{aligned} \tag{27}$$

where $\delta(U)$ denotes to the well known "Dirac" delta function. It is simple to see for $u = U < 0$ the above metric reduces to old one (22).

The injected null matter stress-energy tensor, $T_{\text{matter}}$, can be written before the injection of disturbance into the boundary as (22) which in the Kruskal coordinates become

$$\frac{1}{\kappa}\mathcal{G}_{\{u,v\}} = T_{\text{matter}} = 2T_{uv}dudv + T_{uu}du^2 + T_{vv}dv^2 + T_{ij}d\vec{x}_2^{\,2}, \tag{28}$$

where $\mathcal{G}$ is the Einstein tensor. After injection this tensor could be expressed in the new coordinates such that

$$\begin{aligned} \frac{1}{\kappa}\mathcal{G}_{\{U,V\}} = T_{\text{matter}} &= 2\left(T_{UV} - T_{VV}\alpha\left(X^i\right)\delta(U)\right)dUdV + T_{VV}dV^2 \\ &\quad + \left(T_{UU} + T_{VV}\alpha^2\left(X^i\right)\delta^2(U)\right. \\ &\quad \left. -2T_{UV}\alpha\left(X^i\right)\delta(U)\right)dU^2 + T_{ij}d\vec{x}_2^{\,2}. \end{aligned} \tag{29}$$

By attention to [11, 41] one can consider a massless particle at $u = 0$ which moves in the $v$-direction with the speed of light, the stress-energy tensor of this particle which is corresponds to the shock wave stress-energy tensor is:

$$T_{(shock)UU} = \frac{E}{L^4}e^{(2\pi/\beta)t}a(X)\delta(U), \tag{30}$$

where $E$ is a dimensionless constant and $a(X)$ is a local source of perturbation which for simplicity reasons we take to be as Dirac delta function, i.e. $a(X) = \delta(X)$. By considering the stress-energy tensor of this disturbance the Einstein equation reads $(1/\kappa)\mathcal{G} = T_{\text{matter}} + T_{\text{shock}}$ which should be solved. This equation at the leading order term near the horizon can be solved as follows.

$$\alpha\left(t, x_i\right) \sim e^{-\xi\left(\left|x_i\right| - v_B t\right)}, \tag{31}$$

where,

$$\xi = \sqrt{\frac{f'\left(r_h\right)h'\left(r_h\right)}{2}}, \tag{32}$$

and $v_B$ given by

$$v_B = \frac{2\pi}{\beta\xi} = \sqrt{\frac{f'\left(r_h\right)}{2h'\left(r_h\right)}}. \tag{33}$$

is called as "butterfly velocity". In fact, this velocity as it is mentioned before is the spread of the local perturbation on the boundary of space-time. In our case $h(r) = r^2$ and so the butterfly velocity (33) reads

$$v_B = \sqrt{\frac{\pi T}{r_h}}. \tag{34}$$

Regarding the quintessence dark energy counterpart in the present work we see that the butterfly velocity is depend on the quintessence parameters such as normalization factor $a$ and the state parameter $\omega$. It could be calculated by attention to Hawking temperature $T = f'\left(r_h\right)/4\pi$ as

$$v_B = \frac{1}{2}\sqrt{\frac{1}{r_h^2} + \frac{3}{L^2} - \frac{q_E^2}{r_h^4} + \frac{3a\omega}{r_h^{3\omega+3}}} \tag{35}$$

in which $r_h$ is the outer horizon. To study the action growth in this perturbed geometry two parts must be included: the action of WDW patch behind the (I) past and (II) future horizons. By attention to [3, 22] these two parts are defined respectively by

$$\begin{aligned} \mathcal{S}_{\text{future}} &= \frac{2M}{L\lambda_L}\int \ln e^{\lambda_L\left(\left|t_w\right| - t_* + t_L - \left(|x|/v_B\right)\right)}dx, \\ \mathcal{S}_{\text{past}} &= \frac{2M}{L\lambda_L}\int \ln e^{\lambda_L\left(\left|t_w\right| - t_* - t_R - \left(|x|/v_B\right)\right)}dx, \end{aligned} \tag{36}$$

at which $\lambda_L$ is the Lyapunov exponent proportional to the Hawking temperature and $L$ is the length of the transverse direction $x$. The upper bound of this coordinate called maximal transverse coordinate is $|x| = v_B\left(\left|t_w\right| - t_* - t_R\right)$ that guarantees the emergence of shock wave effect. Time dependence of the action of WDW patch yields:

$$\begin{aligned} \mathcal{S}_{\text{WDW}} &= \mathcal{S}_{\text{future}} + \mathcal{S}_{\text{past}} \\ &= 2M\left(t_L + t_R\right) + 2\mathcal{A}Mv_B\left(\left|t_w\right| - t_* - t_R\right)^2, \end{aligned} \tag{37}$$

where $\mathcal{A}$ is amplitude of shock wave in (31). As we can see by disturbing the geometry the perturbation spreads on the horizon and the action growth get corrected by an extra term which has linear dependence to the speed of perturbation. As the shock wave initially starts from the left side boundary of our two sided black hole and reaches the right side so the extra part depends only on $t_R$. By vanishing any perturbation term $\mathcal{A} \to 0$ one can re-derive non-perturbative situation which has same rate of growth with respect to both $t_L$ and $t_R$.

Now it would be useful to study the effect of dark energy on the butterfly velocity for the same gravity model. As we can see dark energy leads to an extra term to $v_B$ which is addressed as the last term in (35). Since $-1 < \omega < -1/3$ so this term has negative sign. Horizon radius $r_h$ as we know is a solution of $f\left(r_h\right) = 0$. In a charged black hole solution with no dark energy around it we should set $a = 0$ in the equation (8) as

$$\tilde{f}(r) = 1 - \frac{2\widetilde{M}}{r} - \frac{\Lambda r^2}{3} + \frac{\tilde{q}_E^2}{r^2}. \tag{38}$$

and so the corresponding butterfly velocity $\tilde{v}_B$ will be obtain from (35) without the last term and with different horizon radius $\tilde{r}_h$ obtained from $\tilde{f}(\tilde{r}_h) = 0$. From (8) and (38) it is simple to conclude that for fixed mass $M = \widetilde{M}$ and fixed charge $q_E = \widetilde{q}_E$ we have $f(r) < \widetilde{f}(r)$ because $f(r) = \widetilde{f}(r) - a/r^{3\omega+1}$. This equation is situated properly for any horizon radius such as the horizon of quintessence solution, namely $f(r_h) < \widetilde{f}(r_h)$. Since $f(r_h) = 0$ then $\widetilde{f}(r_h) > 0$ and because $\widetilde{f}(\widetilde{r}_h) = 0$ so it leads to $\widetilde{f}(r_h) > \widetilde{f}(\widetilde{r}_h)$. This means the horizon radius of charged black hole solution must be greater when it is surrounded by the dark energy, $r_h > \widetilde{r}_h$. The latter statement can be checked easily as follows: To do so we must be expand Taylor series expansion of the positive function $\widetilde{f}(r_h) > 0$ about the horizon radius in absence of the quintessence dark energy $\widetilde{r}_h$ which leading order term is obtained as

$$\widetilde{f}(r_h) \approx 4\pi\widetilde{T}(r_h - \widetilde{r}_h) + O(2) \tag{39}$$

in which we used the horizon equation of the black hole in absence of the quintessence $\widetilde{f}(\widetilde{r}_h) = 0$ and corresponding the Hawking radiation temperature $4\pi\widetilde{T} = \widetilde{f}'(\widetilde{r}_h)$. Regarding positivity condition on $\widetilde{f}(r)$, $\widetilde{T}$ and $\widetilde{f}(r_h)$ then the equation (39) satisfy the statement $r_h > \widetilde{r}_h$. Substituting $a = 0$ and assuming $\widetilde{q}_E = q_E$ the equation (35) leads to the following form:

$$4\widetilde{v}_B^2 = \frac{1}{\widetilde{r}_h^2} + \frac{3}{L^2} - \frac{q_E^2}{\widetilde{r}_h^4}. \tag{40}$$

for which $\widetilde{r}_h \geq \widetilde{r}_h^{(0)}$ with

$$\widetilde{r}_h^{(0)} = \frac{\sqrt{1 + 12q_E^2/L^2} - 1}{6/L^2}. \tag{41}$$

For small values of charge $qE/L << 1$ the above minimal horizon radius reaches to the following limit.

$$\widetilde{r}_h^{(0)} \longrightarrow |q_E|. \tag{42}$$

In this limit one can write the butterfly velocity (35) as follows.

$$4v_B^2 \approx 4\widetilde{v}_B^2 + \frac{3a\omega}{|q_E|^{3\omega+3}}, \tag{43}$$

where

$$4\widetilde{v}_B^2 = \frac{3}{L^2}. \tag{44}$$

By attention to the conditions $a > 0$ and $-1 < \omega < -1/3$ one can compare (43) and (44) to infer $v_B < \widetilde{v}_B$. It means that the complexity action spreads on the AdS black hole horizon with slower (faster) butterfly velocity in presence (absence) of the quintessence dark energy. In the other side when $\omega$ is decreased and get closer to smallest value $\omega = -1$, then the gap of the butterfly velocity arisen by dark energy get more decreased and for which we have $4v_B^2 \longrightarrow 3/L^2 - 3a$ with $a < 1/L^2$. In short one can infer that by decreasing value of the quintessence state equation parameter the butterfly velocity and so the complexity decreased.

## 4. Concluding Remarks

We studied the complexity growth rate by using "CA" conjecture [2, 3] for a simple model of gravity when its AdS black hole solution is surrounded by quintessence dark energy [30]. The effects of this kind of dark energy is investigated earlier in various works [29–34] and it seems challenging to see how it affects the holographic characteristics. We found some extra terms related to the quintessence dark energy are added to the total action growth. Also it is proved that by attention to the conjugated potential for the quintessence parameter the Lloyd bound [7] is satisfied for all parameter states defined in regime of the quintessence dark energy.

We also investigate the action growth of this model for shock wave geometry [9]. Actually when the boundary is perturbed by a small amount of energy, the geometry in the bulk is affected. The local shock wave spreads near the horizon with the "butterfly velocity" which could be obtained by the equation of motion for the new stress-energy tensor. In fact its form is same as of the old stress tensor but with an extra term which comes from the shock wave and has only $UU$ component. It is due to a massless particle moving at null hypersurface $u = 0$ with the speed of light. We showed that the effect of the quintessence dark energy causes to spread the shock wave with slower butterfly velocity near the horizon, so the complexity growth would be slower as well.

## Data Availability

In fact we do not use Experimental Data in our work. It is free of experimental date and is a purely theoretical research.

## Conflicts of Interest

The authors declare that they have no conflicts of interest.

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