



The Effect of Using Hands-on Materials in Teaching Pythagoras Theorem

Evans Atteh¹, Emmanuel Acquandoh¹, Augustine Boadi¹
and Emmanuel Appoh Andam^{2*}

¹Department of Mathematics, Wiawso College of Education, Sefwi Wiawso Municipal –
Western North Region, Ghana.

²Department of Mathematics, Kumasi Wesley Girls Senior High School, Kumasi, Ghana.

Authors' contributions

This work was carried out in collaboration among all authors. All authors read and approved the final manuscript.

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ABSTRACT

This study sought to improve students' performance in solving problems on the application of the Pythagoras Theorem using hands - on materials. It is an action research and the selected sample of the study was mixed gender class of level 200 (2A1 Class) year group, comprising of forty (40) students in Wiawso College of Education. It involved the use of pre –test and post –test as the methods for data collection. The pre-test was conducted after which an intervention period of four weeks, which involved taking the students through the concept of the application of Pythagoras theorem. A post-test was also conducted after the intervention, the scores obtained from the pre-test and post-test were analyzed by the use of statistical package for social sciences (SPSS). The findings revealed that after the intervention, students were able to overcome the challenges they faced when solving story problems and problems on 3-dimensions involving the Pythagoras Theorem. They could also identify shapes depicting the concept of the Pythagoras' theorem and construction of squares on each side of a right triangle. From the findings it was concluded that the use of hands-on materials in the intervention processes has help improved students understanding and performance.

*Corresponding author: E-mail: andamproject@gmail.com;

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1. INTRODUCTION

Mathematics can be made a fascinating academic discipline with the solving of life like problems whereby students may see its application in the society. Mathematics certainly needs not to be presented to students as dull and uninteresting but then it should be enjoyable and fascinating. Therefore, curriculum objectives need to be built upon what students have learned previously [1]. Thus there are diverse kinds of knowledge that teachers need to know so as to teach mathematics to students [1]. These kinds of knowledge include but not limited to:

- ✓ Pedagogy, such as lesson and unit planning, implementations, questions to use, examples to provide and demonstrations to make situation meaningful and concrete.
- ✓ Content, including important principles and meanings, formulas, processes and procedure, rules, definitions as well as structured ideas.
- ✓ Pedagogical subject matter including determining pre-requisite students' knowledge prior to instruction, Sequence used to develop vital mathematical understanding among learners, diagnosis of students' difficulties in learning as well as the strategies to use with quality materials of instruction.

Most approaches that have grown from constructivism suggest that learning is accomplished best using a hands on approach. Learners learn by experimentation and not by being told what will happen.

The Pythagoras Theorem is a topic in the mathematics syllabus of Ghana at both the Senior High School (SHS) and the Colleges of Education. The syllabus demands that in teaching the topic learners should be taken through practical activities in order to be well abreast with the concept [2]. Learners are supposed to have used the application of the Pythagoras theorem at SHS level. But the students' performance after going through a test conducted involving the application of Pythagoras theorem principles in solving three-dimension problems in Wiawso College of Education was not progressive at all. It became obvious that most learners could not use the knowledge of geometry to determine whether a

triangle is obtuse, acute, or right-angled. The construction of squares on the sides of right-angled triangles forms the basis of activities required to show the relationship between the hypotenuse and the two other sides of the right-angled triangle. On scoring the students, it was realized that they could not answer questions on the application of Pythagoras theorem well. This prompted the researchers to investigate students' problems and to help students acquire knowledge on how to apply the Pythagoras theorem principles.

2. LITERATURE REVIEW

According to [3], Pythagoras' Theorem was developed by the Greek philosopher called Pythagoras and is used in everyday situations. She explains that Pythagoras theorem consists of the following; the sum of the squares of two legs of a right triangle is equal to hypotenuse square. For example if one of the leg is 5 cm and the other leg is 6 cm we can calculate how long the hypotenuse or third leg is, let $a = 5$ $b = 6$ and $c =$ length of hypotenuse then $a^2 = 5^2 = 25$ and $b^2 = 6^2 = 36$. Thus $25 + 36 = 61$ so the square root of 61 is approximately 7.8 cm, therefore we just calculated the length of hypotenuse using this theorem.

In the view of [4], Pythagoras theorem was defined in a triple form. Curtis proposed that a Pythagorean Triple is a set of three positive whole numbers a , b , and c that are the lengths of the sides of a right triangle. This means that a , b , and c satisfy the equation from the Pythagorean Theorem, namely $a^2 + b^2 = c^2$. When " a , b , and c " variables have the greatest common factor of 1, the triangle is called a Pythagorean primitive [4]. He stated that from primitive Pythagorean triples, you can get other, non-primitive ones, by multiplying each of a , b , and c by any positive whole number $d > 1$. This is because $a^2 + b^2 = c^2$ if and only if $(da)^2 + (db)^2 = (dc)^2$. For example, (6,8,10) and (9,12,15) are non-primitive Pythagorean Triples [4]. There are special triangle where the lengths of the three sides have whole number of values for example 3, 4, 5, triangle and 5, 12, 13 triangle together with multiples of these [5]. According to [5,6], three whole numbers such that, the square of one of them is equal to the sum of the squares of the other two, form a Pythagorean triad.

It is believed that there is the need to improve upon teaching and learning in our schools so that

the students and teachers will be well abreast with the right methods in teaching geometry in all levels since the science community accepts geometry [7]. [8] suggested that when teaching geometry, the characteristics and properties of two and three dimensional geometric shapes have to be analysed by the teacher and to develop mathematical arguments about their geometric relationships. Knowledge in geometry can be classified under various themes, such as spatial concepts attributes of 2-dimensional and 3-dimensional shapes, plane geometry, coordinates geometry and deductive geometry [9].

Problem solving approach is considered as very vital in teaching of geometrical concepts [10]. In this approach a problem is posed to learners and through the solution of such a problem they begin to form various geometrical concepts which are inherent in the problem. The problem solving approach also makes use of direct presentation and discovery approaches. The discovery approach is considered as a situation where learners practically manipulate geometry figures to identify their properties as well as using the Cartesian plane to determine images from pre – images [11]. The discovery method is a teaching technique that encourages students to take a more active role in their learning process by answering a series of questions or solving problems designed to introduce a general concept [12]. The discovery method is based on the notions that learning takes place through classification and schema formation [13].

The discovery method is believed to increase retention of material because the student organizes the new information and integrates it with information that has already been stored. It is directly linked to how a teacher should give their students guidance. According to [12] there are three levels of guidance in teaching, these are:

1. **Pure discovering:** The students discovered representative problems to solve with minimal teacher guidance.
2. **Guided discovery:** The students receives problems to solve, but the teacher provides hints and directions about how to solve that problem to keep the student on track.
3. **Expositing:** The final answer or rule is presented to the student.

2.1 Problem Statement

Promoting conceptual understanding is a growing focus in the teaching of mathematics.

There are many ways of teaching the Pythagoras theorem conceptually. But most often, teachers resort to simply giving their students the formula $a^2 + b^2 = c^2$ with little or no reasoning involved. In such a procedural way students' can easily quote the theorem but encounter some difficulties when it comes to its application in solving problems related to situations involving right – angled triangle with different orientation. As a result, most students in Wiawso College of Education do not develop interest towards the study of Pythagoras Theorem and mathematics as a whole.

With sequential investigations, practical activities and problem solving approaches student's confidence level in learning mathematics will be improved; fear and negative attitude towards the subject will be reduced to a minimum level. Students need to experience teachers who possess ample knowledge of subject matter as well as of the method (pedagogy) of teaching mathematics. The authors believe that learners can develop firm conceptual understanding in Pythagoras theorem when hands-on materials are employed sequentially in the mathematics classroom.

2.2 Research Questions

The following questions serve as guide for the study:

1. To what extent does the use of hands – on materials help to improve student understanding of the Pythagoras Theorem and its application?
2. How does the use of hands – on materials increase the ability and capability of students in solving Pythagoras Theorem problems?

2.3 Research Methodology and Design

The design of this study is an action research since it seeks to find solutions to students' inability to solve problems involving the application of the Pythagoras Theorem effectively. An action research however is directed to finding ways of solving practical problem of practitioners. An action research is defined as a process by which practitioners attempt to study their problems scientifically in order to guide, correct and evaluate their decision and actions [14]. It is believed that action research is basically designed to deal with a classroom practice situation [15]. Moreover,

action research is preferred in this context because it deals with a small scale intervention which is appropriate to one classroom situation in which the researcher carried out the study.

2.4 Population and Sampling

The research was conducted at Wiawso College of Education in the Sefwi Wiawso municipality. The school has a population of one thousand and two hundred (1200) students. Four hundred (400) of the students are in the level 200 (second year). The research was conducted in 2A1, a level 200 class which has a population of forty (40) students; the reason for selecting this class is for convenience since the class had just completed learning on Pythagoras theorem and its applications and their exercises showing where the problem exists most. There were sixteen (16) female students and twenty-four (24) male students in that class. The average age of the class was fifteen (15) years and the students came from different regions in Ghana.

2.5 Instrumentation

Pre-test and post-test were used to gather information about the learners' possible cause of low achievement on solving problems involving the application of the Pythagoras Theorem. The instruments were well designed for easy collection, interpretation, analysis and organization of the data collected. The pre-test and post-test were tasks offered to learners to carry out in order to know their level of performance. Again, these tests were used for the evaluation of students' performance before and after the intervention.

2.6 Intervention

There was an intervention with regard to the performance of students after the pre-test concerning the difficulties they had in the application of Pythagoras' theorem principles. This consisted of series of activities (tasks) to determine whether a triangle is acute, right or obtuse and identifying right triangles in 2 and 3-dimensional shapes. In order to raise the interest of the students on Pythagoras Theorem, tasks in the form of practical activities were used by the researchers as the mechanisms to bring the situation under control to the level of the students. The researchers involved the students in demonstration, problem solving and discussion by giving them shapes of drawings to identify right triangles, constructing shape using

mathematical instruments and others. Hardworking students were motivated.

The intervention activities are summed up as follows;

1. An introductory lesson was organized for the students. During this lesson, different varieties of examples and hands-on materials (manipulatives) were used by the researchers to make the lesson more practical.
2. Students were put into an ability groups by the researchers and exercises were given to them to test their understanding. The hands-on materials (manipulatives) were made available to the groups for use.
3. The researchers provided the necessary assistance to the groups where needed.
4. The groups presented their solutions to the exercises given. In situations where the groups had their solution wrong, guidelines were given to the groups to redo the exercises.

3. DATA ANALYSIS AND RESULTS

The results of the study obtained by the students in the pre-test were analyzed and discussed. The pre-test was aimed at finding the knowledge level of students' in applying the Pythagoras Theorem. Several information gathered from the pre-test served as guides to the researcher in developing suitable activities in the form of tasks to assist the learners' to overcome their difficulties. In order to obtain enough data about learners' responses, questions answered were analyzed. In all, seven questions each were given to the students to answer in both the pre-test and the post-test. The questions were marked out of thirty-five (35) marks and were conducted for the forty (40) students and were administered in a period of 45 minutes. Table 1 shows a frequency distribution in percentages of the scores obtained by the students in the pre – test.

After the administration of the pre – test, the authors observed a generally low performance of the students in terms of solving mathematical problems involving the use of Pythagoras theorem and its application, which was below the average score. The indication was that, students were not able to grasp the basic concepts of Pythagoras theorem and therefore they were not able to use appropriate strategies and principles in finding solution to the mathematical problems.

Table 1. Frequency distribution of pre-test scores in percentages

Score	Frequency	Percentage (%)
1-5	12	30
6-10	16	40
11-15	8	20
16-20	4	10
21-25	0	0
26-30	0	0
31-35	0	0
Total	40	100

Table 2. Frequency distribution of post – test scores in percentages

Score	Frequency	Percentages (%)
1-5	0	0
6-10	2	5
11-15	4	10
16-20	6	15
21-25	14	35
26-30	12	30
31-35	2	5
Total	40	100

Table 3. Descriptive statistics of pre – test and post – test

		Paired samples statistics			
		Mean	N	Std. Deviation	Std. Error Mean
Pair 1	pretest	8.7250	40	4.77701	.75531
	posttest	23.0500	40	6.26324	.99031

In order to address the challenges of the students, a series of intervention activities using the hands-on materials for teaching and learning were organized by the authors for the students and a post – test was administered to them.

The post-test was also offered to the students in order to ascertain the effectiveness of the intervention and discussions made on application of the Pythagoras theorem. However, the post-test covered almost all the areas in the pre-test with different questions. Moreover, the responses of students were analyzed and realized from their performances that there has been a tremendous improvement in the student's performance even though all of them did not score 100% in the post-test; the intervention adopted had been very successful. The post-test, involving seven questions were given to the students to answer in the post-test. The questions were marked out of thirty-five (35) marks and were conducted for the forty (40) students and were administered in a period of 45 minutes. Table 2 is the frequency distribution in percentages of the marks obtained by the students in the post – test.

The post – test scores indicated a change in the performance of the students as compared to that of the pre–test scores. The authors attributed the improvement in the students' performance to the use of hands-on materials adopted during the intervention. With the introduction of the hands- on materials, the students were exposed to numerous activities during the intervention processes.

The authors undertook inferential analysis of the pre – test and post – test, and the data used for this analysis were the scores obtained by the students in both tests. Statistical Package for Social Scientist (SPSS) was employed by the authors to obtain the results of the analysis. Table 3 indicates the mean, standard deviation and standard error mean of the paired samples.

The authors therefore concluded that there is a significant difference between the pre – test scores and that of the post–test which is in favour of the post–test. And this is attributed to the intervention processes the researcher took the students through.

4. DISCUSSION

The improvement in performance of the students on the post-test demonstrated much on what was suggested in the literature review. This success came as a result of the authors continuous usage of hands-on materials encouraging different problem solving skills as well as cooperative learning among the students during the intervention activities.

Considering the scores obtained by the students in the pre – test and post – test, as shown in Tables 1 and 2 respectively, it can be confirmed that the performance of the students before the intervention was very abysmal. The respective frequency distribution tables of the pre-test and post-test (Tables 1 and 2) clearly showed the difference in the scores obtained by the students. For instance, in the pre-test (see Table 1), thirty-six (36) out of the forty (40) students scored marks less than half of the 35 marks for the test representing 90% of the total number of students. This poor performance by the students in the pre – test was due to the teaching strategy used in teaching the students and it took the normal rote/lecture form of teaching. With this lecture method of teaching, the students did not have the chance of using their experience to create their own understanding; rather lessons were delivered to them in an organized form previously. The students became used to only memorizing and imitating teachers and this did not help the students to survive independently, applying to Pythagoras theorem as well as real world situations. It was also identified that the students lacked cooperative learning abilities and hence the good students could not help the low performing colleagues. The students could not analyze simple mathematical problems involving Pythagoras theorem and come out with its solution pattern.

Results from the post-test scores by the students, as indicated in Table 2, clearly depict that the students performed much better as compared to the pre-test scores. This suggests that they had improved upon their ability to find solution to mathematical problems involving Pythagoras theorem through the use of hands-on materials. From the frequency distribution of the post – test scores (Table 2), out of the forty (40) students who took part in the test, thirty-four (34) of them obtained either half or more than half of the total mark of 35 for the test, representing 85% of the total students' number. The results from the post-test indicated an upwards trend

which suggests that the intervention activities were effective in assisting the students to overcome their difficulties and helping them in their learning process.

The improvement in the performance of the students, which became evident in the post-test scores they obtained, was not by chance, but through the use of hands-on materials in teaching that the authors employed during the intervention activities. With the use of hand-on materials in teaching, the authors designed a well-planned intervention activity in the lessons with the students. The use of hands-on materials in teaching enabled the students to participate actively in the lessons and also encouraged cooperative learning among the students. And in effect, each student in the group was not only responsible for learning what was being taught alone, but also helped their colleagues who were still having problems and thus created a good learning atmosphere and a co-operative learning spirit amongst them.

5. REVISITING THE RESEARCH QUESTIONS

The results obtained from the post test support the research findings that learning is improved when students are given necessary materials even if they have learning difficulties. The findings from the research study were related to the research questions and assessed whether they had undoubtedly answered the research questions.

The answer to research question 1, “To what extent does the use of hands – on materials help to improve student understanding of Pythagoras Theorem and its application?” The Post-test was offered to the students in order to ascertain the effectiveness of the research question after the intervention and discussions made on application of the Pythagoras Theorem.

Comparing Table 1 and Table 2, showed the result of the intervention by comparing the pre-test scores of the individual students with their respective post test scores. It revealed a significant improvement in students' performance after the introduction of the hands-on materials for teaching and learning by the authors. Table 3 showed the result of the intervention by comparing the pre-test scores of the individual students with their respective post test scores. It revealed a significant improvement in students' performance after the introduction of the hands-

on materials for teaching and learning by the authors. The post-test mean score of 23.05 (Standard deviation of 6.2632) is significantly higher than the pre-test mean score of 8.71 (Standard deviation of 4.7770). This indicated that the use of hands-on material in teaching and learning Pythagoras theorem brought about a tremendous improvement in students' conceptual understanding and performance.

The answer to research question 2, "How does the use of hands – on materials increase the ability and capability of students in solving Pythagoras Theorem problems?"

The authors observed that students who previously employed the chew and pour method of solving mathematical problems involving Pythagoras theorem were discouraged and refrained from such methods. The observations made it clear that students have putting a stop to their usual 'chew 'and 'pour' (memorizing and imitating) method of solving mathematical problems in general. This became evident in the post-test results. From the frequency distribution of the post – test scores (Table 2), out of the forty (40) students who took part in the test, thirty-four (34) of them obtained either half or more than half of the total mark of 35 for the test, representing 85% of the total student's number. This is clear that the use of hands-on materials for teaching through the interventional processes has helped the students to now understand the concept of Pythagoras theorem in solving mathematical problems systematically and not relying on the memorized procedures. With the use of hands-on materials for teaching and learning, the students can have the chance of using their own experience to create their understanding rather than being delivered to them in an already organized form.

6. CONCLUSION

The authors are of the view that if mathematics teachers are able to use the appropriate methods in teaching students such as use of hands-on materials for teaching mathematical concepts which involves:

- ✓ Teachers serving as facilitators and giving students the opportunity in exploring and constructing their own understanding;
- ✓ Using appropriate strategies to present mathematical lessons;
- ✓ Sequencing lessons in a way that will placed the student at the centre of learning;

- ✓ Developing the capacity and the flexibility of thinking that will help students in approaching new areas of mathematics and new problems;
- ✓ Proper understanding and applying of Pythagoras theorem principles;
- ✓ Enjoying mathematics and viewing themselves as capable of doing mathematics.

Then this will bring about meaningful learning for better understanding and not the usual "chew and pour" approach of learning as far as the learning of mathematical concepts is concerned. This suggests that the guaranteed way of making sure students learn mathematics better is when they are well motivated, and the lessons information is presented in well-structured manner using interesting and appropriate teaching and learning strategies and materials, then student will perform better.

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COMPETING INTERESTS

Authors have declared that no competing interests exist.

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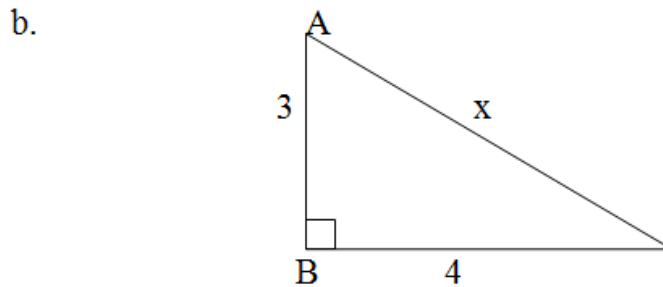
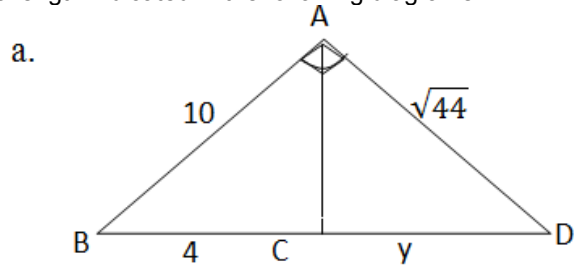
APPENDIX A

Pre-Test Questions Administered to the Students

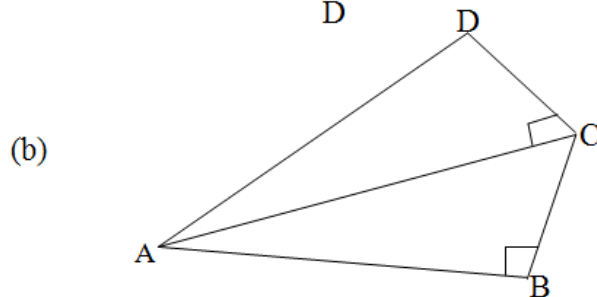
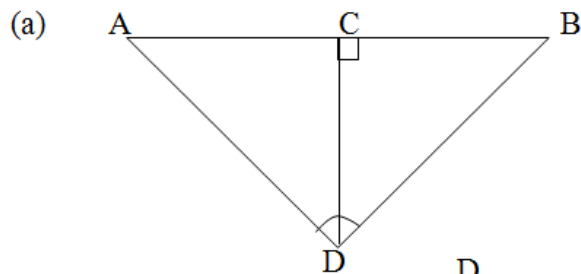
1. The width of a garage is 3.0m, and the roof rises symmetrically to a ridge. The length of one of the sloping roof is 1.7m. The walls are 2.0m high above ground level. Calculate the distance of the highest point of the roof from the floor of the garage.

2. In triangle ABC, angle ABC = 90° |AC| = 12.8cm and |BC| = 8.2cm find the length of |AB|?

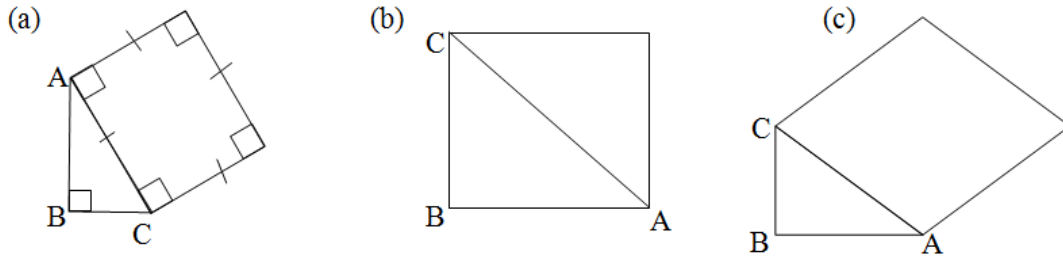
3. Calculate the length indicated in the following diagrams



4. How many right-angled triangles are there in the figures below?



Examine the diagram and state whether it shows a square on the side AC of the triangle ABC away from B



(6) State the equations for the following statements;

a) The square of the hypotenuse of a right-angled triangle equals to the sum of the other two sides squared.

b) The square of the hypotenuse of a right –angled triangle equals to the sum of the squares of the other two sides.

(7) Draw right triangle with $\angle ABC = 90^\circ$ $|AB| = 3\text{cm}$, $|AC| = 5\text{cm}$

a) Find $|BC|$?

b) Construct square on each of its sides

c) What are the similarities of the area of squares drawn?

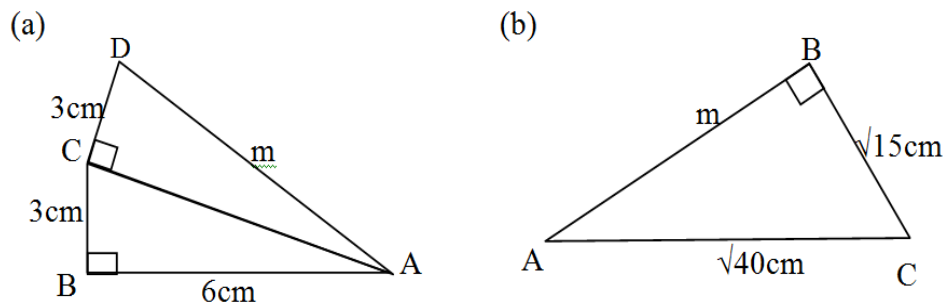
APPENDIX B

Post Test Questions Administered to the Students

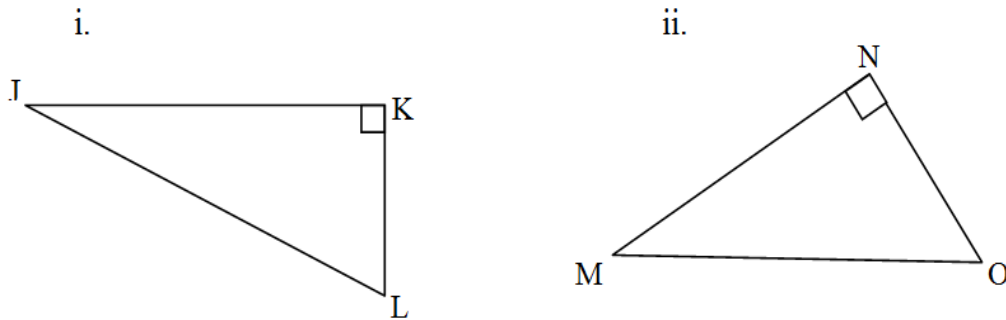
1. A carpenter wanted a flat piece of wood in the shape of a right-angled triangle, the shortest side was to have a length of 5cm. what were the length of the other two sides, if they were whole numbers of centimeters?

2. In triangle XYZ angle $\angle YXZ = 90^\circ$ $|XZ| = a\text{ m}$, $|YZ| = (a+4)\text{m}$ and $|XY| = (a+2)\text{m}$, find a.

3. Calculate the length indicated by the letters m in each of the following diagrams, (Round to 2 d. p)



4a. Copy the following triangles and draw squares on all three sides of each of them.



b. State (any one) equation in which $a^2 + b^2 = c^2$?

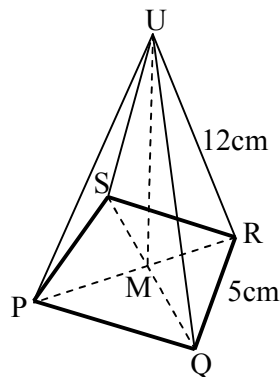
5. Which of the following sets of numbers form Pythagorean triple?

a. {4, 5, 6}

b. {7, 24, 25}

6. A boy placed ladder of length 3m so that one end first reaches the top of a wall, he measured the distance to be 2.5m, calculate the length of the wall.

7. The diagram below shows a square, based pyramid UPQRS in which U is vertically above M, the centre of the base.



a) Find the height of U above the base

b. Identify three triangles which have MU as the perpendicular height.

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