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# Macroeconomic Growth in Vietnam Transitioned to Market: An Unrestricted VES Framework

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Received: 11 June 2020; Accepted: 9 July 2020; Published: 16 July 2020



**Abstract:** The Vietnamese economy has increased at high speed over the transformation decades; however, most recent studies on the economic growth of this country used the Cobb-Douglas or CES (Constant Elasticity of Substitution) production functions, which are unable to explore the relationship between the elasticity of capital-labour substitution and development process, and hence, are not relevant to accessing a dynamic economic system. For that reason, this study is conducted to specify an unrestricted VES (Variable Elasticity of Substitution) production function in a one-sector growth model of Vietnam, highlighted by two characteristics: successful transition from plan to market and rapid progress. The VES is given preference over the CES and the Cobb-Douglas having the elasticity of substitution between capital and labour varying with economic development. By employing a Bayesian nonlinear regression through MCMC methods, the study reported the following findings: (1) the above-unity variable elasticity of capital-labour substitution in an aggregate unrestricted VES function specified for Vietnam shows that the model generates the possibility of endogenous economic growth; (2) the capital share tends to increase, while the labour share faces a downward trend along with the development of Vietnam; (3) the VES is empirically proven through a Bayes factor test to be superior to the CES and Cobb-Douglas for analysis of the growth process of Vietnam, an emerging transition economy.

**Keywords:** Bayesian nonlinear regression; elasticity of substitution between capital and labour; transition; unrestricted VES

## 1. Introduction

As is well known, economic growth is always among the most actual macroeconomic topics, from a theoretical and practical perspective. Indeed, the welfare of subsequent generations will depend on how economic growth currently goes on. Growth theory has attained many achievements, but debates on the origins of economic growth continue. Up to now, the links among economic growth and technical change, the elasticity of capital-labour substitution, and factor shares have been subject to unceasing controversies (for example, [Arrow et al. 1961](#); [Pereira 2003](#); [Miller 2008](#); [Gordon and Vaughan 2011](#)). To perform well-reasoned analysis of this relationship, we should specify an appropriate production function.

Since showing up in 1928, the Cobb-Douglas specification has enjoyed a long-lasting and comfortable life with no serious competitors. Lately, its usefulness, however, has been under doubt. The assumption that the elasticity of substitution between capital and labour and returns to scale are unity is viewed as too restrictive. In particular, the unitary elasticity of factor substitution masks the crucial role of this determinant for economic growth. Growth economists had searched for a more suitable functional form before the emergence of the Constant Elasticity of Substitution (CES) ([Arrow et al. 1961](#)). The CES specification is more flexible but still keeps the elasticity of capital-labour substitution constant, though not necessarily unity. The CES is argued to be inappropriate to elaborate

upon a fast-developing economic system. The Variable Elasticity of Substitution (VES) specification, which allows the factor substitution elasticity to vary with economic development (represented by capital-labour ratio), is more preferable.

The crisis of real socialism in the early 1990s gave rise to a transition process towards economic systems based on market relations. Contrary to the majority of Soviet and East-European transition countries, Vietnam has gained an impressive expansion in production and investment since the beginning of market-oriented reforms in 1986 (Hryeh 2003; Woodruff 2004; Kornai 2006; Mygind 2007). As demonstrated by statistical figures, the growth rate of labour supply is five times lower than that of capital stock in Vietnam (see Table 1), which hypothesizes an increase in the elasticity of factor substitution in response to the rise of capital-labour ratio. It follows that the utilization of the VES specifications for examining Vietnam's economic growth is likely to be reasonable. Despite this, in both foreign and Vietnamese transition literature (for example, Sinelnikov-Murylev et al. 2015; Tu and Nguyen 2012; Nguyen 2013; Khuc and Bao 2016; Pham and Ly 2016; Huynh 2019), the Cobb-Douglas specifications are most often employed, which clearly obstructs insightful investigations in the growth field, where the elasticity of factor substitution is closely associated with economic advancement. Moreover, the application of the Cobb-Douglas or the CES with constant returns to scale often biases estimation results, where the elasticity of factor substitution may be inaccurately estimated (Thach 2020b, 2020c).

**Table 1.** Growth rate of GDP, capital, and labour.

	1990–1999	2000–2009	2010–2019	1990–2019
Real GDP	0.0775713	0.06785719	0.06204859	0.0689003
Capital	0.1211347	0.09988975	0.07306745	0.0970314
Labour	0.024393	0.02039086	0.0125189	0.0192668

Source: The author's calculations.

Also, we desire to stress that the majority of the earlier growth research (for example, Pitchford 1960; Kazuo 1963; De La Grandville 1989; Galor 1995; Klump and Preissler 2000) made use of the old-fashioned frequentist methodology, which might produce unreliable outcomes in many cases (Anh et al. 2018; Kreinovich et al. 2019). This standard approach suffers from some weaknesses, one of which is nonability to provide straightforward and intuitive probabilistic interpretations of estimation results. Its main drawbacks will be discussed later in the current article. On the contrary, Bayesian analysis allows for using probability statements. More importantly, Bayesian results are not point estimates but the full posterior distributions of effects of interest. This reduces model uncertainty.

For the above reasons, the present research focuses on specifying an aggregate unrestricted (nonconstant returns to scale) VES production function for Vietnam on a data sample of 30 years, from which we can derive useful implications for the growth process in emerging transition economies like Vietnam.

The remaining article is structured as follows. Section 2 provides the theoretical background of the Revankar VES production function, followed by empirical studies on this functional form in comparison with the CES and Cobb-Douglas. In Section 3, an aggregate unrestricted VES function for the Vietnamese transition economy is specified through the Bayesian nonlinear regression on a data sample of 30 observations. Section 4 explains Bayesian simulation results, where key estimates are analyzed, and provides a comprehensive discussion on the obtained results. Section 5 presents important conclusions and limitations.

## 2. Conceptual Framework and Empirical Studies

### 2.1. Conceptual Framework

In growth theory, the connections between economic growth and technical change, the elasticity of capital-labor substitution, and factor shares was first conceptually analyzed by Hicks (1932) in his famous Theory of wages. With an unchanged technical level, as the capital stock grows more

rapidly than labour supply, the relative prices of capital and labour decrease, and capital tends to be substituted for labour. More goods and services, production of which requires a large amount of capital, will be produced, and more known but not used earlier capitalistic methods are applied to production. Capital-labour ratio increases until superior applications will run through. Ultimately, the factor substitution elasticity is going to drop below one. At that time, the marginal returns to capital and capital share will decline, while labour share, on the contrary, will increase. The profit rate tends to decrease, and investment is going to run out. The economy eventually reaches the steady state. [Fellner \(1954\)](#) and [Bruton \(1956\)](#) documented that, to counteract a tendency of the marginal productivity and income share of capital to decline as well as the factor substitution elasticity to fall below one, the pace of technical progress should be sufficiently great enough to prevent diminishing marginal productivity of capital from setting in. Moreover, it must be labour-saving to exploit the entire accumulated capital and increase returns to capital. An increase in labour productivity can arise from technical progress and capital formation. Yet, most technical improvements are often embodied in new equipment and machines. It follows that technological progress significantly depends on the growth of investment. Therefore, we need to determine a relevant form of production function to measure types and rate of technical change, to separate the contributions of technical change and capital to total output, and to capture changes in the substitution elasticity resulting from variations in the relative growth rate of labour and capital.

There are a variety of algebraic forms available to describe production functions. Among famous aggregate models, the fixed coefficient function is the simplest form, where the inputs in a fixed proportion, constant returns to scale, and the zero elasticity of substitution are combined in a model. The Harrod-Domar model is a well-known application of this functional form. Since introduced in 1928, the Cobb-Douglas function has come into widespread use due to its simplicity to explain as well as easiness to fit the logarithmic functional form via a normal regression. Nonetheless, this specification is criticized for a too-restrictive assumption that the elasticity of substitution between capital and labour is unity, which is not accordant with real life. As revealed by many empirical analyses, the factor substitution elasticity differs from one (for example, [Pitchford 1960](#); [Nerlove 1967](#); [McFadden 1978](#); [De La Grandville 1989](#); [Azariadis 1993](#); [Galor 1995](#); [Barro and Sala-i-Martin 1995](#); [Duffy and Papageorgiou 2000](#); [Klump and Grandville 2000](#); [Thach 2020a](#)). Moreover, the unitary elasticity of substitution hides a very significant role of it for economic growth. Consequently, [Arrow et al. \(1961\)](#) proposed a CES function with less restrictive assumptions and more fruitfulness, where the value of the factor substitution elasticity ranges from zero to infinite. This is a linear and homogeneous function with constant returns to scale. By assigning appropriate values to the elasticity of factor substitution, the CES can reduce to the Cobb-Douglas or fixed coefficient form. The CES, however, also suffers limitations. The conceptual basis of its formulation is weak. In particular, similar to the fixed coefficient and Cobb-Douglas functions, the elasticity of substitution between capital and labour is constant in the CES. Despite the popularity of the three functional forms, their assumption of a constant elasticity of substitution between capital and labour is too rigid. Growth economists require a more generalized form of production function, where the factor substitution elasticity alters with economic development.

[Revankar \(1971a\)](#) first introduced a standard VES production function. Note here that the CES and the VES have some same properties, except for the case of the constancy of the elasticity of capital-labour substitution along an isoquant for the former, whereas for the latter it remains constant only along a ray from the origin. Completely different from the Cobb-Douglas and the CES, in the VES framework developed in the current study, we could analyze the effect of changes in per capita capital on the factor substitution elasticity. These changes feed back in the economic system, impacting on capital formation and economic growth processes. Under such conditions, the VES model demonstrates the possibility of unbounded endogenous growth even though nonreproducible factors of production are present and exogenous technical progress is absent ([Jones and Manuelli 1990, 1997](#); [Karagiannis et al. 2005](#)). [Grassetti and Hunanyan \(2018\)](#), applying a method to the Solow's type

growth model with differential savings based on a VES function with fixed returns to scale, revealed that, in case of the above-unity elasticity of substitution, an economy featured by production functions with higher elasticity of substitution achieves higher capital and output per-capita equilibrium levels.

Following [Sato and Hoffman \(1968\)](#); [Revankar \(1971a\)](#); [Karagiannis et al. \(2005\)](#), and recently [Thach \(2020a\)](#), in this research, we consider the following VES specification:

$$Q = EK^{\alpha\epsilon}[L + \alpha\beta K]^{(1-\alpha)\epsilon}, \quad (1)$$

where  $Q$ ,  $L$ , and  $K$  represent output, labour input, and capital input, respectively,  $\epsilon$  represents returns to scale,  $E$  denotes efficiency parameter,  $\alpha$ ,  $\beta$  are parameters.

For simplicity, we assume that  $\epsilon = 1$ . Equation (1) is expressed in an intensive form:

$$q = Ek^{\alpha}[1 + \alpha\beta k]^{(1-\alpha)}, \quad (2)$$

where  $q = f(k)$ ,  $q \equiv \frac{Q}{L}$ ,  $k \equiv \frac{K}{L}$ .

The differentiation of Equation (1) results in:

$$f'(k) = \alpha \frac{q}{k} + \alpha(1-\alpha)\beta \frac{q}{1 + \alpha\beta k} \quad (3)$$

The second-order differentiation of Equation (2) yields the following:

$$f''(k) = E\alpha(1-\alpha)(1 + \alpha\beta k)^{-\alpha-1}k^{-1}. \quad (4)$$

$f(k)$  is considered to obtain the properties of a neoclassical function as  $f(k) > 0$ ,  $f'(k) > 0$ , and  $f''(k) < 0 \forall k > 0$ , as long as  $\beta > -1$ ,  $E > 0$ ,  $0 < \alpha \leq 1$ , and  $k^{-1} \geq -\beta$ .

If  $\beta = 0$ , then Equation (2) is specialized to the Cobb-Douglas form, but in case  $\alpha = 1$ , to an Ak type.

The limiting properties of Equation (2) are as follows:

$$\begin{aligned} \lim_{k \rightarrow 0} f(k) = 0, \quad \lim_{k \rightarrow \infty} f(k) = \infty \text{ if } \beta > 0, \\ \lim_{k \rightarrow -\beta^{-1}} f(k) = E(-\beta)^{-\alpha}(1-\alpha)^{1-\alpha} > 0 \text{ if } \beta < 0. \end{aligned} \quad (5)$$

Furthermore, Equation (3) is transformed to:

$$\begin{aligned} \lim_{k \rightarrow 0} f'(k) = \infty, \quad \lim_{k \rightarrow \infty} f'(k) = E(\alpha\beta)^{1-\alpha} > 0 \text{ if } \beta > 0, \\ \lim_{k \rightarrow -\beta^{-1}} f'(k) = E[-\beta(1-\alpha)]^{1-\alpha} > 0 \text{ if } \beta < 0. \end{aligned} \quad (6)$$

Therefore, we have one of the Inada conditions violated if  $\beta > 0$ . This means that the marginal productivity of capital is strictly bounded from below, and labour is not a key input. In other words, in case  $\beta > 0$ , we have:

$$\lim_{L \rightarrow 0} F(K, L) = (\alpha\beta)^{1-\alpha} > 0. \quad (7)$$

Based on Equation (2), the labour share is approximated as follows:

$$\begin{aligned} s_L = \frac{1-\alpha}{1+\alpha\beta k}, \text{ where } \lim_{k \rightarrow 0} s_L = 1-\alpha, \\ \lim_{k \rightarrow \infty} s_L = 0 \text{ if } \beta > 0 \text{ and } \lim_{k \rightarrow -\beta^{-1}} s_L = 1 \text{ if } \beta < 0. \end{aligned} \quad (8)$$

The capital share is expressed by:

$$s_K = 1 - s_L = \frac{\alpha + \alpha\beta k}{1 + \alpha\beta k}. \quad (9)$$

Hence, the elasticity of factor substitution in Equation (2) is expressed by:

$$\sigma(k) = 1 + \beta k. \quad (10)$$

By definition, the elasticity of substitution between capital and labour shows how the ratio of production factors alters when their marginal rate of technical substitution changes by one percent (Thach 2020b). That is, the elasticity of substitution can be regarded as a measure of ease of one factor substituted for another.

To summarize,  $\sigma < 1$  if  $\beta < 0$  and  $\sigma > 1$  if  $\beta > 0$ . In other words, the elasticity of substitution between factor inputs varies with economic progress measured by the per capita capital level. As proven by a large deal of theoretical and empirical studies, the elasticity of factor substitution, in turn, has an impact on the development process. Typically, the findings by Jones and Manuelli (1990, 1997) demonstrate that unbounded endogenous growth can become a reality despite the nonappearance of exogenous technical change and the existence of a nonreproducible factor as long as the marginal productivity of capital is strictly bounded from below. In the meanwhile, Palivos and Karagiannis (2004) affirmed that the elasticity of substitution between factor inputs becoming asymptotically greater than unity (along with the growth of  $k$ ) is necessary and sufficient for the existence of a lower bound on the marginal productivity of capital. Thus, in case  $\sigma > 1$ , along with rising  $k$ , the model generates the possibility of endogenous economic growth.

## 2.2. Empirics on VES

Since the appearance, thanks to the flexible elasticity of factor substitution, the VES production function has been widely accepted by many theoreticians and practitioners in the growth area. The research employing this specification can be categorized into two key groups of models: cross-section and time-series.

Together with (Revankar (1971b); Lu and Fletcher (1968); Lovell (1973a); Kazi (1980); Diwan (1970); Meyer and Kadiyala (1974); Tsang (1976); Zellner and Ryu (1998); Karagiannis et al. (2005)) used cross-section models. Lu and Fletcher (1968), applying an econometric analysis to the time series of 17 two-digit manufacturing industries of the United States, negated the CES supporting the VES in 7 to 9 industries. Revankar (1971b), also relying on the statistical figures from the American manufacturing industries, revealed that the VES specification is more relevant than the Cobb-Douglas to 5 out of 12 industries. Similarly, in the study of Lovell (1973b), owing to the misspecifications of the Cobb-Douglas and the CES, the VES fits the best to data on 3 out of 17 United States two-digit manufacturing industries. Similar to the above works, Kazi (1980) found that the VES is more suitable than the CES in most cases. Using microdata on American two-digit manufacturing companies, Diwan (1970) negated both the Cobb-Douglas and the CES in favour of the VES. Meyer and Kadiyala (1974) achieved the analogous result in a study based on agricultural experimental data. Not aimed at comparing models, the cross-section studies of Tsang (1976), Zellner and Ryu (1998) specified the CES and the VES for the United States transportation equipment and food and kindred products industries, where the substitution elasticity estimates are more than unity. On a sample of 82 countries, Karagiannis et al. (2005) revealed that the elasticity of substitution between capital and labour is greater than one. These findings are not in agreement with Lu and Fletcher (1968) and Kazi (1980), where the factor substitution elasticity is shown to be less than unity.

Time-series models were employed, for example, by Sato and Hoffman (1968); Lovell (1968); Revankar (1971b); Lovell (1973a); Roskamp (1977); Bairam (1989, 1990); Paul (2019); Thach (2020a). Performing an analysis of the private nonfarm sectors of Japan and the United States, Sato and Hoffman (1968) concluded that the VES specification is more relevant than the CES. Through an analysis of the Japanese private nonfarm data, Revankar (1971b) prefers the VES function to the Cobb-Douglas. Slightly



different from the above research, empirical findings by [Lovell \(1973a\)](#) support both the CES and the VES for the United States manufacturing sector. However, the investigation of 16 two-digit American manufacturing industries by [Lovell \(1968\)](#) resulted in the rejection of the Cobb-Douglas and CES forms in favour of the VES. In the works on the Japanese and Soviet economies by [Bairam \(1989, 1990\)](#), the VES is preferred to the Cobb-Douglas. The elasticity of factor substitution is estimated for both the CES and the VES on a sample of 38 Germany manufacturing industries in [Roskamp \(1977\)](#). Recently, [Khan et al. \(2015\)](#) considered the variable elasticity of substitution between production factors the main determinant of growth in the Pakistani banking sector. [Paul \(2019\)](#) applied annual data from 20 Japanese industrial sectors over 43 years (1970–2012) and announced an empirical finding favouring a choice of a VES framework. As a further step in this regard, [Brianzoni et al. \(2012\)](#); [Grasseti and Hunanyan \(2018\)](#) elaborated upon the connection between elasticity of factor substitution and growth, while assuming the VES function within the Solow framework, while [Michetti \(2013\)](#) accessed the standard Solow–Swan growth model with a sigmoidal production function. These studies indicated that fluctuations and complex dynamics may emerge in case the elasticity of factor substitution is sufficiently small. Based on a corporate data set for the period 2008–2018, a VES function was specified to test the conditions of endogenous growth in the manufacturing sector of Vietnam ([Thach 2020a](#)). Notably, the majority of the mentioned studies resulted in the substitution elasticity estimates of less than unity, except for [Bairam \(1989\)](#); [Roskamp \(1977\)](#); [Thach \(2020a\)](#).

Remarkably, most previous studies on the VES functions employed the outdated frequentist methods. Moreover, our literature review conducted above demonstrates conflicting findings: the elasticity of factor substitution obtains varying estimates, which explores a complicated relationship between capital and labour.

### 3. Methodology and Data

#### 3.1. Estimation Method and Model

Since the 1990s, Bayesian analysis has become a commonly used approach in behavioural and social science investigations in the context of the current crisis in classical econometrics (see [Nguyen and Thach 2018, 2019, 2020b, 2020c](#); [Anh et al. 2018](#); [Nguyen et al. 2019a, 2019b](#); [Sriboonchitta et al. 2019](#); [Svítek et al. 2019](#); [Kreinovich et al. 2019](#); [Tuan et al. 2019](#); [Thach et al. 2019](#)). First, the Bayesian framework has a great advantage in that Bayesian statistical inference does not yield point estimates as in frequentist analysis, but a full distribution function of a particular effect. So, the Bayesian approach permits such probability statements as the effect is unlikely or likely. We can specify the probability of one or some regression variables having negative (or positive) effects rather than a simple statement that a coefficient or other is either significant or not. So, Bayesian methods solve the problem of nonsignificant, but potentially substantial, effects not being reported ([Cohen 1994](#); [Starbuck 2006](#)). Second, universality is another main superiority of Bayesian analysis over the frequentist approach. The Bayes rule is applicable to all regression models, while a frequentist method designed for a certain model class cannot often be applied to other model classes. Third, Bayesian posterior distributions are more balanced due to combining prior knowledge, belief and theory with available data. Especially, weakly prior distributions recommended for use in Bayesian modeling allows for regularizing parameter estimates, whereas noninformative priors might incur type I and type M errors ([Lemoine 2019](#)). This middle perspective in the Bayesian inference can mitigate the effect of a small sample size, produce more balanced, robust and reliable results.

In the present work, we used the Bayesian nonlinear regression to estimate an aggregate unrestricted VES production function for Vietnam in three stages of its transformation to a market-oriented economy. In Vietnam, the market-oriented reforms began with the first transition stage from 1990 to 1999. In the second transition stage between 2000 and 2009, Vietnam entered the phase of fast export-led growth. Since 2010, this economy has experienced economic recovery and post-crisis growth. These stages are delimited by the Asian financial crisis and the great global

recession. In growth literature, the Bayesian nonlinear regression is relevant to accessing growth processes (see, for example, Davidian and Giltinan 1995; Pinheiro and Bates 2000). To examine the nature of the Vietnamese economic growth, we chose a VES function instead of a CES or Cobb-Douglas. That is because the Vietnamese economy has undergone rather rapid growth in production and investment during the transformation decades. As a result, capital-labour interaction might vary in a rapidly developing economy. According to statistical data (Federal Reserve Bank of St. Louis 2019; General Statistics Office of Vietnam 2008, 2019), the average growth rate of real GDP, capital stock and labour supply over 30 years achieved 6.9, 9.7, and 1.9 percent, respectively, in Vietnam. If each of the transition stages is considered, the above indicators are equivalent to 7.8, 12.1, and 2.4 percent for 1990–1999, 6.8, 10, and 2 percent for 2000–2009, 6.2, 9.7, and 1.3 percent for 2010–2019 (Figure 1, Table 1). On average, capital increases five times faster than labour during the transformation period. The higher growth rate of capital in comparison with that of labour is assumed to have affected the capital-labour ratio, which in turn has triggered changes in the elasticity of factor substitution.

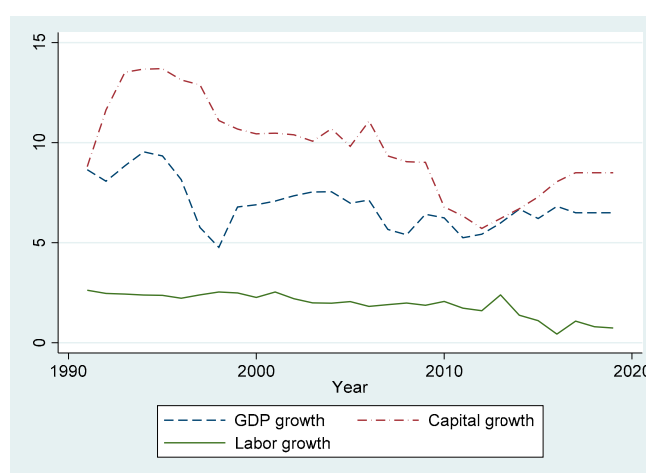


Figure 1. Main aggregate growth rates. Source: The author’s calculations.

Additionally, another advantage of the VES specification over the CES and Cobb-Douglas is that the VES enables returns to scale to alter. Hence, we used an unrestricted VES function in this work, that is, with nonconstant returns to scale ( $\epsilon \neq 1$ ). Based on Equation (1), we performed three Bayesian simulations in a similar way for three transition stages of Vietnam, where owing to the skewed distributions of the variables, the nonlinear functions need to be transformed to the logarithmic form. Primarily, it must be noted that prior choice is essential in Bayesian modeling. As mentioned earlier, weakly informative priors are considered to have regularizing effects on posterior distributions. Thus, advocating the middle view, we assign weakly informative priors to the parameters of interest  $\alpha, \epsilon, \beta$ . Since the coefficient  $\beta$  might take a negative sign, contrast to Thach (2020a), we pick a  $N(0,1)$  prior on it. Concerning the remaining parameters of less importance, we set noninformative prior distributions. Our models look like:

Likelihood:

$$\ln gdp = b_0 + \alpha \epsilon \ln K + (1 - \alpha) \epsilon \ln(L + \alpha \beta K), \tag{11}$$

Priors:

$$\begin{aligned} b_0 &\sim N(0, 100) \\ \epsilon &\sim N(0, 1) \\ \alpha &\sim \text{uniform}(0, 1) \\ \beta &\sim N(0, 1) \\ \text{sig}_{2_0} &\sim \text{Igamma}(0.001, 0.001), \end{aligned} \tag{12}$$

where  $\ln gdp, \ln K, L,$  and  $K$  represent natural log of output, natural log of capital, labour, and capital, respectively.

To prove empirically that the VES is the most appropriate for accessing growth process in the transformation period of Vietnam, we estimate the CES and the Cobb-Douglas functions based on the same data sample. For each of the functions, we perform Bayesian simulations. Further, we conduct a Bayes factor test to verify which function is the best. The CES function used has such same form as introduced in [Arrow et al. \(1961\)](#):

$$F(K, L) = \gamma(\alpha K^{-\rho} + (1 - \alpha)L^{-\rho})^{\frac{-1}{\rho}} \quad (13)$$

where  $\rho = \frac{1-b}{b}$  is substitution parameter,  $\alpha = a_1 \times \gamma^\rho$  is distribution parameter;  $\gamma$  is efficiency parameter, the elasticity of factor substitution,  $\sigma = \frac{1}{1+\rho}$ .

We use the traditional Cobb-Douglas function:

$$F(K, L) = AK^\delta L^{1-\delta}, \quad (14)$$

where  $A$  is efficiency parameter,  $\delta$  and  $1 - \delta$  is the elasticity of output with respect to capital and labour, respectively.

For convenience to estimate, we transform the nonlinear production functions to log–log forms.

### 3.2. Data Description

This study used annual time series for the 30-year period from 1990 to 2019 on real GDP, stock of capital, and labour supply of Vietnam to estimate an aggregate VES specification for national production with nonconstant returns to scale (unrestricted VES). Data on GDP and stock of capital at constant 2011 prices were obtained from the Penn World Tables ([Federal Reserve Bank of St. Louis 2019](#)). The database of the ([General Statistics Office of Vietnam 2008, 2019](#)) provides us with the number of members in the labour force. GDP and stock of capital are expressed in billions of U.S. dollars and units of labour are millions of workers.

## 4. Research Results

### 4.1. Descriptive Statistics

Table 2 provides descriptive statistics on the variables GDP, K, and L. They have a maximum value of 734,854.3, 2,227,366, and  $5.72 \times 10^7$ , a minimum value of 106,419.8, 151,858.5, and  $3.29 \times 10^7$ , and a mean of 342,900.7, 881,396.3, and  $4.58 \times 10^7$ , respectively. Standard deviation is equivalent to 185,052.5, 615,966.8, and 7,822,043 for GDP, K and L, respectively.

**Table 2.** Descriptive statistics.

Variable	Obs	Mean	Std. Dev.	Min	Max
Real GDP (gdp)	30	342,900.7	185,052.5	106,419.8	734,854.3
Capital (K)	30	881,396.3	615,966.8	151,858.5	2,227,366
Labour (L)	30	$4.58 \times 10^7$	7,822,043	$3.29 \times 10^7$	$5.72 \times 10^7$

Source: The author's computations.

### 4.2. Bayesian Simulation Results

In this subsection, the estimation results of the aggregate unrestricted VES function corresponding to the three transition stages of Vietnam will be analyzed. The model summaries in Table 3 report that (1) posterior mean and median as commonly used point estimators in Bayesian analysis are similar in our case, indicating that posterior distributions are symmetric; (2) for the parameters  $\alpha$ ,  $\varepsilon$ , and sig2\_0, the credible intervals do not contain zero in the three simulation results and thus, we can claim that capital, labour, and returns to scale have strongly positive effects on the outcome; accordingly, probability of a positive mean of the parameter sig2\_0 is 100 percent, probability of the parameters



$\alpha$  and  $\varepsilon$  belonging to the interval (0,1) is 100 percent and 97–99 percent, respectively; (3) regarding the parameter  $b_0$ , strongly positive effects occur in two first simulations as zero does not fall into the credible intervals; probability of a positive mean ranges from 69 percent to 100 percent; (4) concerning the parameter  $\beta$ , though in two of the three simulations, the credible intervals do not encompass zero, probability of a positive mean is very high, varying from 91 percent to nearly 100 percent; and, (5) more importantly, for all the parameters, the estimates of standard deviation are low, whereas those of Monte-Carlo standard error (MCSE) are close to one decimal. Both these indicators point at the high accuracy of the mean estimates.

**Table 3.** Bayesian nonlinear simulations.

Parameter	Transition Stage	Mean	Std. Dev.	MCSE	Median	Probability of Mean >0	Equal-Tailed [95% Cred. Interval]
alpha	1990–1999	0.7004347	0.2064727	0.01541	0.739583	1 *	0.2774297, 0.987269
b0		1.237738	0.3356832	0.048401	1.312661	0.99	0.3298835, 1.6785
beta		0.7798368	0.6317611	0.049588	0.7062168	0.91	−0.085491, 0.149432
epsilon		0.7174294	0.1035089	0.01572	0.6879733	0.97 **	0.6080182, 0.014937
sig2_0		0.0009322	0.0006678	0.000015	0.0007618	1	0.000317, 0.0026863
alpha	2000–2009	0.5996914	0.2338747	0.042459	0.583051	1 *	0.2167813, 0.9784832
b0		1.115017	0.186176	0.023994	1.137348	1	0.6282, 1.426625
beta		0.7794099	0.608169	0.068298	0.6240513	0.98	0.00798062, 0.303366
epsilon		0.7520055	0.0742376	0.015827	0.7319812	0.99 **	0.6639818, 0.9548205
sig2_0		0.0004123	0.0002971	$9.0 \times 10^{-6}$	0.0003354	1	0.0001424, 0.0011015
alpha	2010–2019	0.6190902	0.221302	0.041054	0.6059595	1 *	0.2574442, 0.9827137
b0		0.1127431	0.2717961	0.029972	0.118131	0.69	−0.4590531, 0.633356
beta		0.9196118	0.5319415	0.036116	0.8015926	0.999	0.1480791 2, 0.192505
epsilon		0.8833604	0.0478443	0.00907	0.8800785	0.98 **	0.7983294, 0.9996468
sig2_0		0.0004738	0.0004458	0.000015	0.0003676	1	0.0001611, 0.0013634

Note: \*  $0 < \text{probability of mean} \leq 1$ , \*\*  $0 < \text{mean} < 1$ . Source: The author's computations.

In a MCMC framework, Bayesian estimation results should be robust to be applied to inference. For this, we should check for the convergence of the MCMC algorithm. Once the convergence of MCMC sequences is established, the model parameters will converge to some reasonably fixed values. There are a lot of convergence tests available for use: visual and formal. We applied both in this work.

First, a graphical test via cusum plots was performed. The cusum plots allow for judging how well the sequences mix. The worse the mixing of the sequences, the smoother the cusum lines. Conversely, the better the mixing of the sequences, the more jagged the cusum lines. Regardless of how long the mixing of sequences may take, if the sequences mix sufficiently well, then the joint posterior distribution will be sufficient to extract marginalized distributions through the integration of nuisance parameters. Figure 2a–c exhibits the jagged cusum lines corresponding to the parameters of the Bayesian simulations. Namely, each of the subfigures a–c describes cusum plots for all the model parameters (alpha,  $b_0$ , beta, epsilon, and sig2\_0) of the simulation related to the respective transitional stage.

Second, formal tests like effective sample size were accessed; the key indicators, such as acceptance rate and efficiency impact on the chain convergence too. As shown in Table 4, regarding the Bayesian simulations relative to three transition stages, the acceptance rate ranges from 0.33 to 0.45, which is reasonable according to the criteria suggested by Roberts and Rosenthal (2001), while the simulation efficiency of all the parameters is higher than the critical level of 0.01. Corr. time is small enough to accept all the efficiencies.

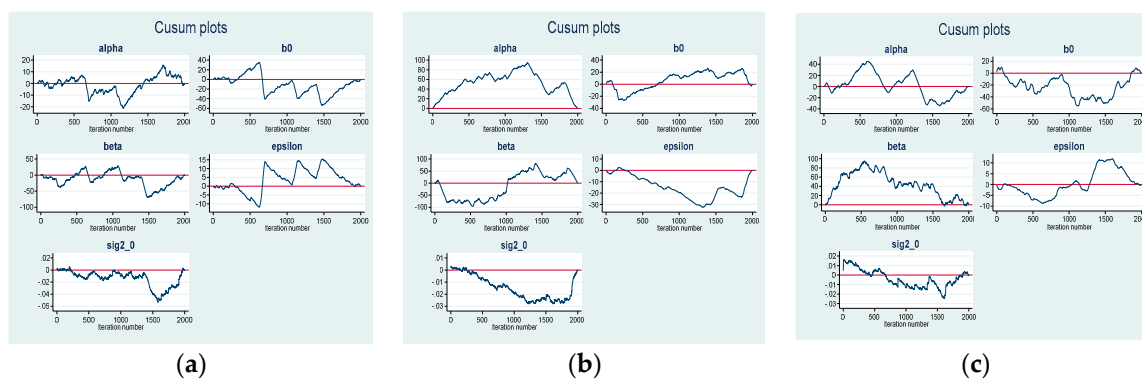


Figure 2. Cusum plots of model parameters of three simulations (a–c). Source: The author’s computations.

Table 4. Formal tests for MCMC convergence.

Parameter	Transition Stage	Acceptance Rate	Efficiency	Corr. Time
alpha	1990–1999	0.45	0.0898	11.14
b0			0.0241	41.58
beta			0.0812	12.32
epsilon			0.0217	46.13
sig2_0			1.0000	1.00
alpha	2000–2009	0.35	0.0152	65.92
b0			0.0301	33.22
beta			0.0396	25.22
epsilon			0.0110	90.90
sig2_0			0.5398	1.85
alpha	2010–2019	0.33	0.0145	68.83
b0			0.0411	24.32
beta			0.1085	9.22
epsilon			0.0139	71.87
sig2_0			0.4719	2.12

Source: The author’s computations.

### 4.3. Model Comparison

To prove that the VES is preferable to the Cobb-Douglas and the CES for the transformation period of Vietnam, we estimated these functions on data covering the entire interval of time and then performed a Bayes factor test to find the best function. The estimation results are presented in Table 5. In general, the less the Deviance Information Criterion (DIC) estimate, the higher log(ML) and log(BF) values, the better the model. In sum, the VES estimates demonstrate the best results.

Table 5. Comparison between the VES, CES, and Cobb-Douglas models via a Bayes factor test.

	DIC	log(ML)	log(BF)
Cobb-Douglas	−107.1396	38.21688	-
CES	−105.3526	35.84787	−2.36901
VES	−291.276	53.21536	14.99848

Source: The author’s computations.

#### 4.4. Discussion

According to the simulation results presented in Table 2, all the estimated mean coefficients are different from zero and economically plausible. Mean coefficients for  $\alpha$  are 0.59, 0.6, and 0.7. Importantly, the mean coefficients for  $\varepsilon$  are estimated to be close to unity ( $\varepsilon = 0.72, 0.75, 0.88$ ). These findings are consistent with mild decreasing returns to scale in the Vietnamese economy. Except for (Karagiannis et al. 2005), we have found no empirical studies where an unrestricted VES function is analyzed. Our study is the first to specify an unrestricted VES function for a transition economy.

More interestingly, the main finding concerning our tested hypothesis is that the sign of the mean coefficients estimated for  $\beta$  is positive for all the simulations ( $\beta = 0.78, 0.78, 0.92$ ). The estimates seem rather small at first glance but surely exert a potentially great influence on the elasticity of substitution between capital and labour, taking into account Equation (10):  $\sigma(k) = 1 + \beta k$ . Hence, our estimation results indicate that the variable elasticity of factor substitution  $\sigma$  generally is higher than unity for all the transition stages of Vietnam. The estimation of the unrestricted VES function for the entire period 1990–2019 yields the similar result, that is, a variable elasticity of factor substitution is found to be higher than unity too. Our findings are in agreement with the predictions of endogenous growth theory (Romer 1986, 1987), where accumulation of capital in a broad sense is positively associated with sustainable economic growth, with applied growth theories such as Jones and Manuelli (1990, 1997); Palivos and Karagiannis (2004); Grasseti et al. (2018), as well as a great number of empirical studies (e.g., see, Bairam 1989; Roskamp 1977; Khan et al. 2015; Grasseti et al. 2018; Thach 2020a), in case  $\sigma > 1$ , along with rising  $k$ , the model generates the possibility of endogenous economic growth. These findings imply that Vietnam generated the possibility of endogenous growth in the three stages as well as in the entire period of the market-oriented transformation. Evidence on spillover effects contributing to endogenous economic growth in Vietnam may be as follows: learning-by-doing effects, impact of human capital, effect of technical progress embodied in physical investment, intensification of R&D activities, increasing trade openness. However, according to the results of model comparison performed through a Bayes factor test (DIC, log(ML), log(BF)) by the author, the VES specification is preferable to the CES and the Cobb-Douglas at the country-data level.

Notably, referring to (8) and (9), the labour share and the capital share depend on the estimates of  $K$ ,  $L$ ,  $\alpha$  and, most importantly,  $\beta$ . As the estimates of the coefficient  $\beta$  are positive and different from zero, the factor shares change with the capital-labour ratio. Our estimation results point out that the capital share is likely to grow significantly over time. These findings are in line with the statistic figures (General Statistics Office of Vietnam 2008, 2019) and the suggestions of growth theories, i.e., capital accumulation results in a constant increase in the capital share, while the labour share decreases.

Last, since the mean estimates of  $\beta$  are found to be positive, the variable elasticities of capital-labour substitution are more than unity. It follows that the capital and labour shares vary with economic development. The results allow us to reject empirically the Cobb-Douglas and the CES specifications in favour of the VES in the context of Vietnam's transition economy, where growth of capital stock has so far outperformed that of labour supply over several decades.

## 5. Conclusions

In this work, we specified an aggregate unrestricted VES production function in a one-sector growth for the Vietnamese economy, which has experienced a transformation period consisting of three stages. In contrast to frequentist methods resulting in point estimates, the Bayesian nonlinear regression utilized to estimate the VES function yields the entire posterior distribution of the coefficients of interest, thereby minimizes model uncertainty and increases its robustness. In the application of MCMC methods, convergence diagnostics need to be conducted, and the test results show that the MCMC chains have converged to the desired distribution. The empirical results demonstrate that the estimates of the variable elasticity of substitution between capital and labour are higher than unity, implying that the Vietnamese economy in general exhibits the possibility of endogenous growth during the transition period, in which the rate of capital accumulation has much outperformed growth rate of

labour supply under the condition of technical progress. In fact, the rise of the key factors contributing to technical progress and endogenous growth such as Foreign direct investment (FDI), domestic investment, technology transfers, human capital accumulation, institutional reforms is observed in the transformation period of Vietnam. One more significant empirical finding of our study is that the VES specification is proven through a Bayes factor test to be more appropriate in comparison with the CES and Cobb-Douglas to represent production technology in fast-growing economies like Vietnam. These results are consistent with a satisfactory description of data used in the research, applied economic theories, and the predictions of endogenous growth theory.

Based on the obtained empirical results, some policy implications are proposed. To achieve the real possibility of endogenous growth, Vietnam must combine capital accumulation with enhancement of technical progress. Unceasing technical progress counteracts a tendency of diminishing marginal returns to capital. For this purpose, this country needs to strongly encourage investments in physical and human capital, reinforce R&D activities, as well as expand technology transfers. Despite rapid investment growth in Vietnam over the last 30 years, private capital now accounts for a minor share in the aggregate capital and should be encouraged more, while public capital must be reduced to a necessary minimum. Domestic firms should more enhance technology transfers, raise production capabilities, and carry out their own R&D. A comprehensive and selective policy of FDI attraction ought to be focused on high-quality investments disseminating positive spillover effect to domestic firms. Accumulation of human capital needs to be extensively intensified in many ways. Besides, it is necessary to overcome the negative consequences left from a centrally administered economy such as ineffective state-owned enterprises and the state's excessive interference in the economic processes. So, importantly, overall institutional reforms should be implemented effectively. Furthermore, a reasonable policy of international integration considerably contributes to economic growth. The mentioned policy implications for Vietnam are useful for both transition and developing countries.

The primary limitation of the current research may be that a restricted VES function for aggregate production has not been specified. Returns to scale can theoretically be unity, that is, returns to scale are probably constant. Hence, future studies are motivated to investigate both unrestricted and restricted VES production functions.

**Funding:** This research is supported by the Banking University HCMC, Vietnam.

**Acknowledgments:** The author is thankful to the anonymous referees for very useful comments.

**Conflicts of Interest:** The author declares no conflict of interest.

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